# SOLUTIONS OF DELAY EQUATIONS IN BANACH SPACES

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## ABSTRACT

The aim of this talk is to show two characterizations of well-posedness for the equation

$$u'(t) = Au(t) + Fu_t + f(t), \qquad t \in \mathbb{R}, \tag{1}$$

where $(A, D(A))$ is a closed linear operator on a Banach space $X$, $u_t(\cdot) = u(t+\cdot)$ on $[-r, 0], r > 0$, and the delay operator $F$ is supposed to belong to $\mathcal{B}(L^p([-r, 0], X), X)$ for some $1 \le p < \infty$ or $\mathcal{B}(C([-r, 0], X), X)$.

First studies on equation (1) goes back to J. Hale and G. Webb. A general and systematic study of linear delay equations with emphasis on the qualitative behavior and asymptotic properties can be found in the recent monograph by Bátkai and Piazzera [1]. The problem to characterize the well-posedness of (1) arises naturally from recent studies on maximal regularity and their application to nonlinear problems in the theory of evolution equations, see the recent monograph by Denk-Hieber-Prüss [2] and references therein. Our main tool to solve the problem is the connection between differential equations and operator-valued Fourier multipliers as noticed in [3, 4].

We first show necessary and sufficient conditions in order to obtain existence and uniqueness of periodic solutions for equation (1) in the periodic Lebesgue spaces $L^p(\mathbb{T}, X), 1 < p < \infty$ (see [5]). In contrast with earlier papers on the subject, we do not assume that $A$ generates a $C_0$-semigroup. Instead, our results involves $UMD$-spaces and $R$-boundedness, which are not too restrictive conditions for applications concerning nonlinear problems.

In the second part, we are able to obtain necessary and sufficient conditions in order to guarantee well-posedness of the delay equation (1) in the Hölder spaces $C^\alpha(\mathbb{R}, X) \quad (0 < \alpha < 1)$, and under the condition that $X$ is a $B$-convex space (see [6]). We stress that also here $A$ is not necessarily the generator of a $C_0$-semigroup and that now, in contrast with the case of Lebesgue spaces, the condition of $R$-boundedness is not required.

## References

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