VIII Congreso de Análisis Funcional y Ecuaciones de Evolución

Dedicado al 60° aniversario de Manuel Pinto

Actas de Comunicaciones 2013

Prefacio

Este Octavo congreso organizado por el Grupo de Análisis Funcional y Ecuaciones de Evolución, mejor conocido por su sigla GAFEVOL, está dedicado al conocimiento y al crecimiento. El conocimiento, que compartimos en esta reunión, pues nos actualiza y nos motiva a continuar trabajando en nuestra pasión: La Matemática.

El crecimiento, pues originalmente el grupo estaba conformado por investigadores del Departamento de Matemática y Ciencia de la Computación de la Universidad de Santiago y, gracias a las actividades de difusión de este pequeño grupo, tales como la organización de congresos nacionales e internacionales y coloquios semanales, entre otros, contribuyeron a ir sumando año a año nuevos(as) colaboradores(as). Muchos de ellos(as), son actualmente ex-alumnos(as) de la USACH, que comenzaron participando tímidamente como asistentes a coloquios semanales o en las sesiones invitadas en los Congresos Nacionales e Internaciones de Ecuaciones de Evolución. Hoy, se desempeñan como académicos en distintas universidades de nuestro continente y continúan colaborando con el grupo mediante sus investigaciones y acciones vinculadas a él. Este crecimiento ha permitido generar redes de colaboración entre los académicos vinculados al grupo, lo que además de enriquecer el trabajo científico, ha permitido continuar con la formación de nuevos estudiantes.

Asimismo, mencionamos aqui un desafio importante que deseamos propiciar para el nuevo año que enfrentamos. Este desafio consiste en la apertura hacia los estudiantes de pregrado de nuestra área. Es asi como destacados estudiantes que también nos acompañan en este Congreso, podrán el día de mañana - mediante la proacción de todos los expositores presentes en este congreso - ser nuestros colegas de investigación y hacer crecer la actividad científica que realizamos, para que podamos soñar que en un futuro próximo seamos los líderes indiscutidos de ésta área de la matemática tanto en nuestro País como en la Región.

Mención aparte merece el reconocimiento que deseamos hacer en este Octavo congreso a un destacado matemático, el profesor Manuel Pinto de la Facultad de Ciencias de la Universidad de Chile, quien ha sido un lider en el campo de su especialidad, con más de 400 citaciones y 100 publicaciones internacionales de primer nivel, actualmente realiza una de las mayores contribuciones al área con sus conocimientos e ideas. Por esta razón, celebramos en esta ocasión su sexágesimo aniversario y le rendimos un sentido homenaje.

Comité Organizador

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Comité Organizador

- Carlos Lizama, Universidad de Santiago de Chile.
- Hernán Henríquez, Universidad de Santiago de Chile.
- Rodrigo Ponce, Universidad de Talca.
- Gonzalo Robledo, Universidad de Chile.
- Sergei Trofimchuk, Universidad de Talca.

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- Eduardo Hernández, Universidade de São Paulo, Ribeirão Preto, Brasil.
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- Mahamadi Warma, Universidad de Puerto Rico, Rio Piedras, USA.

Conferencias

Existence and uniqueness of wavefronts for a non-local reaction-diffusion equations with distributed time delay.

Maitere Aguerrea

Abstract

The main object of study is the non-local reaction-diffusion equation

$$u_t(t,x) = u_{xx}(t,x) - f(u(t,x)) + \int_0^\infty \int_{\mathbb{R}} K(s,w) g(u(t-s, x-w)) dw ds, \qquad (1)$$

where the time $t \geq 0$, $x \in \mathbb{R}$, the kernel K satisfies $K \in L^1(\mathbb{R}_+ \times \mathbb{R})$, $K \geq 0$ and $\int_0^\infty \int_{\mathbb{R}} K(s,w) dw ds = 1$. Here, K can be asymmetric. We are interesting in the study of semi-wavefront solutions of equation (1), i.e. bounded positive continuous non-constant waves $u(t,x) = \phi(x+ct)$, propagating with speed c, and satisfying the boundary condition $\phi(-\infty) = 0$. An important special case of semi-wavefront is a wavefront, i.e. positive classical solution $u(t,x) = \phi(x+ct)$ satisfying $\phi(-\infty) = 0$ and $\phi(+\infty) = \kappa$. We establish the existence and uniqueness of semi-wavefronts solutions in the non-degenerate case. In fact, the existence and uniqueness results are proved for all speeds $c \geq c_\star$, where the determination of $c_\star$ is similar to the calculation of the minimal speed of propagation. Finally, we apply our results to some non-local reaction-diffusion epidemic and population models with distributed time delay.

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Proportional-Differential Equations.

William Campillay and Manuel Pinto*

Abstract

Newtonian calculus is constructed on the basis of the operations: $+$, and $\cdot$. If we change the addition by multiplication and the product by: $a \otimes b = a^{ln(b)}$ we can consider the field $(\mathbb{R}^+, \cdot, \otimes)$ where $\mathbb{R}^+$ is the set of positive real numbers. The absolute value would correspond to the relative value $[[x]] = x$ if $x \geq 1$ ó $[[x]] = x^{-1}$ if $x \in (0,1)$ for which $[[x]] \leq p$ if only if $p^{-1} \leq x \leq p$. In this world, the proportionality operates as the basic form of comparison and the operation division is replaced by: $a \oslash b = a^{\frac{1}{ln(b)}}$. Consider that e is neutral element for $\otimes$:

$$a \otimes e = a, \quad a \oslash e = a.$$

The $\otimes$-inverse of a is $a^{\{-1\}} := e^{\frac{1}{ln(a)}}$, since $a \otimes a^{\{-1\}} = a^{\{-1\}} \otimes a = [e^{\frac{1}{ln(a)}}]^{ln(a)} = e$. Considering the following equivalence $a \oslash b = e$ if and only if $a = b$, we incorporate a way to compare positive quantities. Thus, the proportional derivative is defined comparing the proportions $\frac{f(x)}{f(x_0)}$ with $\frac{x}{x_0}$, through the operation $\oslash$:

$$\widetilde{f}(x_0) = \lim_{x \to x_0} \frac{f(x)}{f(x_0)} \oslash \frac{x}{x_0}.$$

It is easy to interpret and understand the geometric meaning of this derivative, for instance, if $\widetilde{f} \geq 1$, then f increases and:

$$\widetilde{f \cdot g} = \widetilde{f} \cdot \widetilde{g}, \quad \widetilde{f \otimes g} = [\widetilde{f} \otimes g] \cdot [f \otimes \widetilde{g}] \;\; (Product\ Formula),$$

$$\widetilde{f \circ g} = \widetilde{f(g)} \otimes \widetilde{g} \quad (Chain\ Rule).$$

Naturally the proportional integral that satisfies the fundamental theorem of calculus for the proportional derivative is built. In this opportunity the corresponding differential equations and some of their meanings are set. The bounded solutions of the non-homogeneous proportional-differential equation :

$$\widetilde{x(t)} \cdot [\alpha(t) \otimes x(t)] = \beta(t).$$

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are studied, where $\alpha, \beta : (0, \infty) \to (0, \infty)$.

This Calculus has been applied in several situations [2,3] in the study of Lorenz System [1], analysis of biomedical image [4],etc.

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An inverse problem in biological olfactory cilium.

Carlos Conca

Abstract

In this lecture we study a linear inverse problem with a biological interpretation, modelled by a Fredholm integral equation of the first kind, where the kernel is represented by step functions. Based on different assumptions, identifiability, stability and reconstruction results are obtained.

Partially supported by Basal-CMM project and Fondecyt 1130317, Department of Mathematical Engineering, Center for Mathematical Modelling, University of Chile, Santiago, e-mail: cconca@dim.uchile.cl

Modeling and simulation of the response to a treatment for metastasis to the liver of a gastrointestinal stromal tumor.

Patricio Cumsille Atala

Abstract

At its most essential level, cancer involves the abnormal growth and spread of tissues within a body. What occurs at the nano-scale of molecules and micro-scale of cells affects the behavior of tissue at the centimeter-scale - and vice versa. In order to better understand these multi-scale linkages, mathematical modeling, analysis, and simulation have been employed to study tumor behavior [10].

Modeling was developed in oncology not only in order to understand and predict tumor growth, but also to anticipate the effects of targeted and untargeted therapies. Exemples of such models are [9, 7, 6, 11, 5, 2]. More complex multi-scale models have been set up to address questions in the prediction of efficacy of chemotherapy and radiotherapy [3, 4, 12], and to investigate the effect of cytostatic treatments such as anti-angiogenic and anti-maturation therapies [13, 12, 8]. Thus, a very wide range of these models exist, involving many stages in the progression of tumors. Few models, however, have been proposed to reproduce in vivo tumor growth because of the complexity of the mechanisms involved [1].

In this work we introduce a new mathematical model to simulate metastasis to the liver of a gastrointestinal stromal tumor (GIST), focusing in the study of two important issues: tumor growth and efficacy of mixed strategies (anti-proliferative and anti-angiogenic drugs) targeted to cancer. Examples of such drugs used in the treatment of metastasis of a GIST are the Glivec and the Sutent respectively. We assume that Glivec acts like a chemotherapy: it gives back apoptosis in the cell cycle, while that Sutent acts like an anti-angiogenic drug, with no cytotoxic effects. By the other hand, clinical experience shows that most cases of metastasis to the liver of a GIST behaves similarly, which may characterized in the three following phases: phase 1 is characterized by a good answer to Glivec, phase 2 by a resistance to Glivec, which produces an uncontrolled tumor growth. Despite of this fact, in phase 2 the tumor shows a good answer to Sutent, whose action controls again the tumor growth. Finally the phase 3 is characterized by a resistance to Sutent, which produces again an uncontrolled tumor growth. Thus, in order to simulate adequately these

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three phases, we assume that there are three kind of proliferating tumor cells: the first ones which are sensitive to the two kinds of drugs, the second ones which are sensitive only to the anti-angiogenic drug, and the third ones which are not sensitive to any of the drugs. This is the main novelty of our modeling. The simulations we have carried out show good agreement with experimental data obtained from patients reached with malignant GIST, as will be shown in the talk.

This work has been carried out in the frame of a research stay of Patricio Cumsille at Institut de Mathématiques de Bordeaux, Université de Bordeaux I, in collaboration with members of INRIA MC2 team.

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Oscillatory and periodic solutions in impulsive differential equations with a general piecewise constant argument.

Kuo-Shou Chiu

Abstract

We prove the existence and uniqueness of the solutions of a class of first order nonhomogeneous impulsive differential equations with a general piecewise constant argument. We also study the conditions of periodicity, oscillation, nonoscillation and global asymptotic stability for some special cases.

Let $\mathbb{Z}$, $\mathbb{N}$ and $\mathbb{R}$ be the set of all integer, natural and real numbers, respectively. Fix two real-valued sequences t_i, γ_i, $i \in \mathbb{N}$, is a strictly ordered sequence of real number, such that $t_i \le \gamma_i \le t_{i+1}$, $t_i \to \infty$ as $i \to \infty$. Let $\gamma : [0,\infty) \to [0,\infty)$ be a step function given by $\gamma(t) = \gamma_i$ for $t \in I_i = [t_i, t_{i+1})$.

In this talk we study the nonhomogeneous impulsive differential equations with general piecewise constant arguments (IDEPCAG)

$$\begin{cases} y'(t) + a(t)y(t) + b(t)y(\gamma(t)) = f(t), \quad y(\tau) = y_0, \\ \Delta y_i|_{t=t_i} = J_k(y(t_i^-)), \ k \in \mathbb{Z}, \end{cases} \tag{2}$$

where $a(t)$, $b(t)$, $f(t)$ and J_k are real-valued continuous functions of t defined on $\mathbb{R}$, $\gamma(t) = t_i$ if $t_i \le t < t_{i+1}$ and $\Delta y(t_i) = y(t_i) - y(t_i^-)$, $y(t_i^-) = \lim_{h\to 0^-} y(t_i + h)$.

Also we will investigate the equation

$$\begin{cases} y'(t) + a(t)y(t) + b(t)y(\gamma(t)) = 0, \quad y(\tau) = y_0, \\ \Delta y_i|_{t=t_i} = J_k(y(t_i^-)), \ k \in \mathbb{Z}. \end{cases} \tag{3}$$

Differential equations with piecewise constant argument (DEPCA) represent a hybrid of continuous and discrete dynamical systems and therefore combine the properties of both the differential and difference equations, See [3,11,12]. These equations have many applications in the control theory and certain biomedical models [2]. Theory and practice of DEPCA of generalized type have discussed extensively in [1,4-7,9,10].

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The research of first order differential equations with piecewise constant arguments of delay and advanced type (mixed type) was initiated by Cooke and Wiener [3] and Shah and Wiener [11]. A lot of results concerning the scalar versions of equation (2) without impulsive effects have been studied in [6,7] and the references cited therein. To the best of our knowledge, there are only a few papers involving impulsive differential equations with piecewise constant arguments [8,13].

Our goal in this talk is to derive the sufficient conditions for the existence of solutions of equations (1) and (2), and periodicity, oscillation, nonoscillation and global asymptotic stability for equation (2).

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Propiedad de l^p-acotación para la ecuación en diferencia de Volterra.

Claudio Cuevas, Mario Choquehuanca, Filipe Dantas and Herme Soto*

Abstract

En este trabajo presentamos resultados sobre la existencia de l^p-soluciones para la ecuación lineal en diferencia de Volterra tipo convolución de la forma

$$u(n+1) = \lambda \sum_{j=-\infty}^{n} a(n-j)u(j) + f(n), \ n \in \mathbb{Z}, \tag{4}$$

donde λ es un número complejo, $a : \mathbb{N} \to \mathbb{C}$ es una función sumable y f está en $l^p(\mathbb{Z}, X)$.

Para un determinado $\lambda \in \mathbb{C}$, sea $s(\lambda, k) \in \mathbb{C}$ la solución de la ecuación en diferencia

$$s(\lambda, k+1) = \lambda \sum_{j=0}^{k} a(k-j)s(\lambda, j), \ k = 0, 1, 2, \ldots, \ s(\lambda, 0) = 1. \tag{5}$$

En este caso, $s(\lambda, k)$ es llamada la solución fundamental (4) generada por $a(\cdot)$. Definamos el conjunto
$\Omega_s := \{\lambda \in \mathbb{C} : ||s(\lambda, \cdot)||_1 := \sum_{k=0}^{\infty} |s(\lambda, k)| < +\infty\}$.

Teorema 1 *Sea λ en Ω_s. Entonces para cualquier $f \in l^p(\mathbb{Z}, X)$ la ecuación (4) tiene una única solución $u(\cdot)$ en $l^p(\mathbb{Z}, \mathbb{X})$ la cual es dada por:*

$$u(n+1) = \sum_{j=-\infty}^{n} s(\lambda, n-j)f(j). \tag{6}$$

La solución $u(\cdot)$ satisface $u \in l^{p'}(\mathbb{Z}, \mathbb{X})$ para todo $1 \leq p \leq p' \leq \infty$, y la siguiente estimación se cumple:

$$||u||_{l^{p'}(\mathbb{Z},\mathbb{X})} \leq ||s(\lambda, \cdot)||_1^{1-\frac{1}{p}+\frac{1}{p'}} ||s(\lambda, \cdot)||_\infty^{\frac{1}{p'}} ||f||_{l^p(\mathbb{Z},\mathbb{X})}. \tag{7}$$

En particular, si $p = \infty$, obtenemos

$$||u||_\infty \leq ||s(\lambda, \cdot)||_1 ||f||_\infty. \tag{8}$$

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Self-similarity, symmetries and asymptotic behavior in Morrey spaces for a fractional wave equation.

Marcelo Fernandes de Almeida

Abstract

This paper is concerned with a fractional PDE that interpolates semilinear heat and wave equations. We show results on global-in-time well-posedness for small initial data in the critical Morrey spaces and space dimension $n \geq 1$. We also remark how to derive the local-in-time version of the results. Qualitative properties of solutions like self-similarity, antisymmetry and positivity are also investigated. Moreover, we analyze the asymptotic stability of the solutions and obtain a class of asymptotically self-similar solutions.

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Poincaré's problem in the class of almost periodic type functions.

Pablo Figueroa and Manuel Pinto*

Abstract

We consider the Poincaré's classical problem of approximation for second order linear differential equations in the class of almost periodic type functions. We obtain an explicit form for solutions of these equations by studying a Riccati equation associated with the logarithmic derivative of a solution. The fixed point Banach argument allows us to find almost periodic and asymptotically almost periodic solutions of the Riccati equation. A decomposition property of the perturbations induces a decomposition on the Riccati equation and its solutions. In particular, by using this decomposition we get asymptotically almost periodic and also p-almost periodic solutions to the Riccati equation.

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Monotone Traveling Waves for Delayed Reaction-Diffusion Equations .

*S. Trofimchuk and A. Gomez**

Abstract

In this work we consider the problem of existence of monotone traveling wavefronts solutions $u(t,x) = \phi(x \cdot \nu + ct)$, $\phi(-\infty) = 0$, $\phi(+\infty) = \kappa > 0$, $\|\nu\| = 1$ of the family of reaction-diffusion equations with delay $h \geq 0$,

$$u_t(t,x) = \Delta u(t,x) + f(u(t,x), u(t-h,x)),$$

where f is a $C^{1,\gamma}$-smooth function and $g(x) = f(x,x)$ is a `monostable function`: $g(0) = g(\kappa) = 0$, $g'(0) > 0, g'(\kappa) < 0$ and $g(x) > 0$ for all $x \in (0,\kappa)$.
This family includes some classical equations as KPP-Fisher delayed equation ($f(x,y) = x(1-y)$) and the diffusive Nicholson's Blowflies equation ($f(x,y) = -x + \frac{p}{\delta} y e^{-y}$). The proof of the main theorem uses the Liapunov-Schmidt reduction over suitable weighted C^2-smooth function spaces.

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Strict solutions for abstract neutral differential equations.

Eduardo Hernández

Abstract

We present some recent results on the existence of strict solutions for abstract neutral differential equations of the form

$$u'(t) = Au(t) + f(t, u_t, u'_t), \quad t \in [0, a], \tag{9}$$

$$u_0 = \varphi \in \mathcal{B}, \tag{10}$$

where $A : D(A) \subset X \to X$ is the generator of an analytic semigroup of linear operators $(T(t))_{t\geq 0}$ defined on a Banach space $(X, \|\cdot\|)$, $f : [0, a] \times \mathcal{B} \times \mathcal{B}_X \to X$ is a suitable continuous function, $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$ and $(\mathcal{B}_X, \|\cdot\|_{\mathcal{B}_X})$ are abstract phase spaces defined axiomatically, u_t is the history of $u(\cdot)$ at the time t and the symbol u'_t denotes the derivative at t of the function $s \to u_s$.

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Continuous Dependence for Impulsive Functional Dynamic Equations involving Variable Time Scales.

M. Bohner, M. Federson and J. G. Mesquita

Abstract

Using a known correspondence between the solutions of impulsive measure functional differential equations and the solutions of impulsive functional dynamic equations on time scales (see [1, 2]), we prove that the limit of solutions of impulsive functional dynamic equations over a convergent sequence of time scales converges to a solution of an impulsive functional dynamic equation over the limiting time scale. More precisely, we prove that the solutions of

$$\begin{cases} x(t) = x(t_0) + \displaystyle\int_{t_0}^{t} f(x_s, s)\,\Delta s + \sum_{\substack{k\in\{1,\dots,m\},\\ t_k<t}} I_k(x(t_k)), & t \in [t_0, t_0+\eta]_{\mathbb{T}_n}, \\ x(t) = \phi(t), \quad t \in [t_0 - r, t_0]_{\mathbb{T}_n} \end{cases} \tag{11}$$

converges uniformly to the solution of the problem

$$\begin{cases} x(t) = x(t_0) + \displaystyle\int_{t_0}^{t} f(x_s, s)\,\Delta s + \sum_{\substack{k\in\{1,\dots,m\},\\ t_k<t}} I_k(x(t_k)), & t \in [t_0, t_0+\eta]_{\mathbb{T}}, \\ x(t) = \phi(t), \quad t \in [t_0 - r, t_0]_{\mathbb{T}} \end{cases} \tag{12}$$

whenever $d(\mathbb{T}_n, \mathbb{T}) \to 0$ as $n \to \infty$. Here, $d(\mathbb{T}_n, \mathbb{T})$ denotes the Hausdorff metric.

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Funciones Remotamente Casi Periódicas y ecuaciones diferenciales.

Christopher Maulén and Manuel Pinto*

Abstract

Realizaremos una introdución a las funciones remotamente casi periódicas, definidas por D. Sarason en [1] en 1984, demostrando que son un sub-álgebra cerrada de las funciones continuas generada por las funciones casi periódicas y las funciones lentamente oscilantes. Estas funciones han sido poco estudiadas, principalmente por C. Zhang et al., en los siguientes trabajos [3], [4] y [5], y además D. Piao et al. en [6]. Comenzaremos presentando ejemplos, propiedades y problemas básicos. Luego, compararemos dichas funciones con las funciones casi periódicas [2], y asintóticamente casi periódicas [7].
Además, estudiaremos la existencia de soluciones remotamente casi periódicas para sistemas de ecuaciones diferenciales

$$x' = A(t)x(t) + f(t).$$

Con A y f funciones remotamente casi periódica, usando dicotomía exponencial, algúnas técnicas del análisis y algúnas condiciones suficientes obtendremos la existencia y unicidad de soluciones del tipo remotamente casi periódico para estos sistemas. Veremos la importancia y necesidad de la condición de bi-remotamente casi periodicidad, del correspondiente nucléo de Green en la obtención de las soluciones.

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Evolución de un espín central en un baño de espines nucleares.

Jeronimo Maze

Abstract

La coherencia de estados cuánticos electrónicos y nucleares es de vital importancia en la implementación de sistemas nanoscópicos como sensores de alta sensitividad y memorias de información cuántica. En este charla presentaré sobre las propiedades de coherencia de un grado de libertad cuántico de espín relacionado con el defecto nitrógeno-vacante en el diamante. Este espín presenta buenos tiempos de coherencia del orden de milisegundos a temperatura ambiente. Veremos que esta decoherencia esta limitada por la interacción de este espín central con un baño de espines nucleares. Se discutirá sobre la dinámica nuclear y posibles formas de extender los tiempos de coherencia.

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Two different dynamical systems, and how Manuel Pinto helped me in their study.

Alejandro Omón Arancibia

Abstract

In this talk I will give some results on two very different dynamical systems. The first one is the Parabolic Gelfand Problem, this is a reaction diffusion PDE with exponential nonlinearity. The main results to present are concern with the relation between the initial condition and the development (or not) of blow-up. Very specific relations on whether blow-up is developed or not in term of the initial conditions are given, the same as estimations of blow-up time and eventually blow-up profiles. A very dynamical system approach is used. In this frame, also interesting information on steady states is given.

The second part of the talk concerns the nonlinear Navier-Stokes system. This is a problem that has a long history in Applied Mathematics, and it is still an enormous source of ideas in Mathematics. The main focus is given first to study some reported solutions, and after this there are given some regularity results in terms of Pressure, one of the unknowns of the problems, that in general is not studied in deep.

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On a class of abstract neutral functional differential equations.

*Eduardo Hernández, Michelle Pierri and Andréa Prokopczyk**

Abstract

In this work, we study the existence of mild solutions for a class of neutral functional differential equations of the form

$$\frac{d}{dt}[x(t)+g(t,x_t)] = Ax(t)+f(t,x_t), \qquad t\in[0,a], \tag{13}$$

$$x_0 = \varphi\in\Omega\subset\mathcal{B}, \tag{14}$$

where $A : D(A)\subset X\to X$ is an almost sectorial operator, $(X,\|\cdot\|)$ is a Banach space, $\mathcal{B}$ is the phase space ($\mathcal{B}=C([-r,0],X)$ or $\mathcal{B}=L^p([-r,0],X)$), $\Omega\subset\mathcal{B}$ is open and $g,f:[0,a]\times\Omega\to X$ are suitable functions.

There exists a extensive literature on abstract neutral differential equations treating the case in which A is a sectorial operator, see for example [1, 2, 3, 4] and the references therein. Sectorial operators appears frequently in applications since many elliptic differential operators are sectorial when they are considered in Lebesgue spaces (L^p-spaces) or in spaces of continuous functions, see [5].

However, if we look at spaces of more regular functions such as the spaces of Hölder continuous functions, we find that these elliptic operators are not sectorial, see [5, Example 3.1.33] and [6]. Nevertheless, for these operators estimates such as

$$\|(\lambda-A)^{-1}\|\le\frac{M}{|\lambda|^{1-\alpha}},\quad \lambda\in\Sigma_{\omega,\theta}=\{\lambda\in\mathbb{C}:\ |\arg(\lambda-\omega)|<\theta\},$$

with $\alpha\in(0,1)$, $\omega\in\mathbb{R}$ and $\theta\in(\frac{\pi}{2},\pi)$ are available (see [6]), which allows to define an associated "semigroup" by means of the Dunford integral

$$T(t)=\frac{1}{2\pi i}\int_{\Gamma_\theta}e^{\lambda t}(\lambda-A)^{-1}d\lambda,\quad t>0,\quad (\Gamma_\theta=\{te^{i\theta}:\ t\in\mathbb{R}\setminus\{0\}\}).$$

Under the above conditions, the operator A is called almost sectorial and the operator family $\{T(t),T(0)=I:\ t\ge 0\}$ is said the semigroup of growth α generated by A. The

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semigroup $(T(t))_{t\geq 0}$ has properties similar at those of analytic semigroup which allows to study some classes of partial differential equations via the useful methods of semigroup theory.

As an application of our results we can consider the partial neutral differential equation concerning the heat conduction in fading memory material. In the classical theory of heat conduction, it is assumed that the internal energy and the heat flux depends linearly on the temperature u and on its gradient ∇u. Under these conditions, the classical heat equation describes sufficiently well the evolution of the temperature in different types of materials. However, this description is not satisfactory in materials with fading memory. The next system, see [7, 8], has been frequently used to describe this phenomena,

$$\begin{aligned} \frac{d}{dt}\left[u(t,x) + \int_{-\infty}^{t} k_1(t-s)u(s,x)ds\right] &= c\triangle u(t,x) + \int_{-\infty}^{t} k_2(t-s)\triangle u(s,x)ds, \\ u(t,x) &= 0, \quad x \in \partial\Omega, \end{aligned}$$

where $\Omega \subset \mathbb{R}^n$ is open, bounded and has smooth boundary, $(t,x) \in [0,\infty) \times \Omega$, $u(t,x)$ represents the temperature in x at the time t, c is a physical constant and $k_i : \mathbb{R} \to \mathbb{R}$, $i = 1, 2$, are the internal energy and the heat flux relaxation respectively.

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Soluciones casi automórficas para la ecuación en diferencia de Volterra con retardo infinito.

Herme Soto and Airton Castro*

Abstract

Sea $\mathbb{X}$ un espacio de Banach. En este trabajo presentamos existencia de solución discreta casi automorfica de la siguiente ecuación funcional de Volterra semi-lineal en $\mathbb{X}$

$$u(n+1) = \lambda \sum_{j=-\infty}^{n} a(n-j)u(j) + f(n, u_n), \ n \in \mathbb{Z}, \tag{15}$$

donde $u_n : \mathbb{Z}_- \to \mathbb{X}$ es la función historia, la cual es definida por $u_n(\theta) = u(n+\theta)$ para todo $\theta \in \mathbb{Z}_-$, donde λ es un número complejo, $a(n)$ es una función sumable $\mathbb{C}$-valuada y f es una perturbación no lineal no necesariamente globalmente Lipschitz. Los resultados son consecuencia de aplicaciones de diferentes teoremas de punto fijo, tales como, principio de la contracción, de Leray-Schauder y de Krasnoselskii.

Para establecer uno de los principales resultados, necesitamos introducir las siguientes condición y definiciones:

$\mathcal{B}$ denota el espacio de fase, el cual se define axiomáticamente y en el cual se satisface:

(B) Si $(\varphi^n)_{n\in\mathbb{N}}$ es una sucesión acotada uniformemente en $\mathcal{B}$ la cual converge puntualmente a φ, entonces $\varphi \in \mathcal{B}$ y $\|\varphi^n - \varphi\|_{\mathcal{B}} \to 0$ cuando $n \to \infty$.

$(H1)$ Suponga que $f : \mathbb{Z} \times \mathcal{B} \to \mathbb{X}$ es localmente Lipschitz con respecto a la segunda variable, esto es, para cada número positivo σ, para todo $k \in \mathbb{Z}$ y para todo $\varphi, \psi \in \mathcal{B}$ con $||\varphi||_{\mathcal{B}} \leq \sigma$ y $||\psi||_{\mathcal{B}} \leq \sigma$, tenemos $||f(k,\varphi) - f(k,\psi)|| \leq L_f(\sigma)||\varphi - \psi||_{\mathcal{B}}$, donde $L_f : \mathbb{R}^+ \to \mathbb{R}^+$ es una función no decreciente.

Para un determinado $\lambda \in \mathbb{C}$, sea $s(\lambda, k) \in \mathbb{C}$ la solución de la ecuación en diferencia

$$s(\lambda, k+1) = \lambda \sum_{j=0}^{k} a(k-j)s(\lambda, j), \ k = 0, 1, 2, \ldots, \ s(\lambda, 0) = 1. \tag{16}$$

Definimos el conjunto $\Omega_s := \{\lambda \in \mathbb{C} : ||s(\lambda, \cdot)||_1 := \sum_{k=0}^{\infty} |s(\lambda, k)| < +\infty\}$.

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Detección de un obstáculo inmerso en un fluído usando mediciones en la frontera.

*Jaime Ortega and Sebastián Zamorano**

Abstract

En este trabajo estudiamos el problema inverso asociado al sistema de Stokes. Sea $\Omega \subset \mathbb{R}^n$ un conjunto abierto con frontera $\partial\Omega$ lo suficientemente suave. El problema consiste en determinar un objeto D inmerso en Ω tomando mediciones de la velocidad del fluido en movimiento y de las fuerzas en la frotera. Es decir, el siguiente problema:

$$\begin{cases} -\mathrm{div}(\sigma(u,p)) = 0 \quad , & \text{en } \Omega \setminus \overline{D}, \\ \mathrm{div} u = 0 \quad , & \text{en } \Omega \setminus \overline{D}, \\ u = g \quad , & \text{sobre } \partial\Omega, \\ u = 0 \quad , & \text{sobre } \partial D, \end{cases} \tag{17}$$

donde $\sigma(u,p) = \mu(\nabla u + \nabla u^T) - pI$, I es la matriz identidad de $n \times n$ y μ es la viscosidad.

Sea η el vector normal a $\partial\Omega$. Así, el problema a resolver es intentar recuperar D con $g \in H^{1/2}(\partial\Omega)$ y tomando mediciones, en $\partial\Omega$, de las componentes normales del tensor

$$\sigma(u,p) \cdot \eta = \psi.$$

Bajo hipótesis sobre la frontera de Ω, de clase Lipschitz, y de regularidad sobre g en la frontera de Ω, se tiene unicidad para el problema inverso descrito, lo cual fue probado, usando técnicas de continuación única, por Alvarez et. al. [2].

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