PERIODIC SOLUTIONS OF SECOND ORDER DIFFERENTIAL EQUATIONS IN BANACH SPACES

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Abstract. We are concerned with the second order abstract differential equation:

$$\begin{cases} u''(t) - Au(t) &= f(t), \qquad 0 \le t \le 2\pi \\ u(0) &= u(2\pi), \\ u'(0) &= u'(2\pi), \end{cases} \tag{0.1}$$

in the spaces $L^p_{2\pi}(\mathbb{R}; X), 1 \le p < \infty$ (where X is a Banach space, $a\,\alpha \in \mathbb{R}$ and $A : D(A) \subset X \longrightarrow X$ is a closed linear operator.

In the first part, we discuss recent results on operator valued Fourier multiplier theorems in the spaces $L^p((2\pi); X)$. These involve the notion of UMD spaces and $R-$boundedness of operator families. Then, we consider study mild (generalized) solutions for the above second order problem. Two types of mild solutions are considered. When the operator A involved is the generator of a strongly continuous cosine function, we give characterizations in terms of Fourier multipliers and spectral properties of the cosine function.

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2000 *Mathematics Subject Classification.* Primary 47D06, 47D09; Secondary 47F05, 35L70, 35G20.

The first author is supported in part by Convenio de Cooperación Internacional (CONICYT-Chile) Grant # 7010675 and the second author is partially financed by FONDECYT Grant #1010675 .